

15PH3008 Mathematical Physics II

Set B

Time : 3 hrs
Total Marks: 100

1.

(a) Let $f(z) = \frac{1}{z} = u(x, y) + iv(x, y)$. (15)

- a) Show explicitly that the Cauchy-Reimann conditions are satisfied.
b) Show explicitly that both real part and imaginary part satisfy Laplace equations.

(b) Find the value of the integral $\int_0^{1+i} z^2 dz$. (5)

OR

2.

(a) Evaluate the integral $\oint_C f(z) dz$ for

$$f(z) = \frac{5z - 2}{z(z - 1)}$$

where C is along the circle $|z| = 2$ in the counterclockwise direction. (a) Use Cauchy's residue theorem. (b). Use the second residue theorem. (15)

(b) State Cauchy's Residue Theorem . (5)

3.

(a) Obtain Fourier's series for the expansion $f(x) = x \sin x$ in the interval $-\pi < x < \pi$.
Hence deduce that

$$\frac{\pi}{4} = \frac{1}{2} + \frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots \quad (15)$$

(b) Find the finite Fourier cosine transform of x. (5)

OR

4.

(a) Find the finite Fourier sine transform and finite Fourier cosine transform of $\frac{\partial^2 U}{\partial x^2}$ where U is a function of x and t for $0 < x < L$, $t > 0$. (15)

(b) Find the value of $\sum_{n=1}^{\infty} \frac{1}{n^2}$ using Fourier's Series. (5)

5.

(a) Evaluate $I = \int_0^1 \frac{1}{1+x} dx$ correct to 3 decimal points by Simpson's 1/3 rule. (15)

(b) Given $\frac{dy}{dx} = 1 + y^2$ where $y=0$ when $x=0$. Find $y(0.2)$. (Runge-Kutta IV order). (5)

OR

6. (a) Find the solution of inhomogeneous differential equation $\frac{d^2\psi}{dx^2} = f(x)$ subject to the homogeneous boundary conditions $\psi(0)=\psi(1)=0$. (15)
- (b) Write down the basic properties of Green's function. (5)
7. (a) Prove that $(Z_5, +_5)$ forms an Abelian group. (15)
- (b) For any group $(G, *)$, prove that the identity element is unique. (5)

OR

8. (a) Prove that the two left cosets aH and bH of a subgroup H of a group G are either identical or disjoint. (15)
- (b) Prove that the center of a group G is a subgroup of G . (5)
9. (a) The number of students in a prestigious central university for every ten years starting from 1965 was as given below. Estimate the number of students in that university for the years 1969 and 1999. (15)
- | | | | | | |
|----------------------------|------|------|------|------|------|
| Year | 1965 | 1975 | 1985 | 1995 | 2005 |
| Number of students (x 100) | 46 | 66 | 81 | 93 | 101 |
- (b) Using Central Difference Formula (Gauss's Forward Formula), find the value of $e^{2.17}$ from the following table. (5)

| | | | | | | | |
|------------------|--------|--------|--------|--------|--------|--------|--------|
| x | 2.00 | 2.05 | 2.10 | 2.15 | 2.20 | 2.25 | 2.30 |
| y=e ^x | 7.3891 | 7.7679 | 8.1662 | 8.5849 | 9.0250 | 9.4877 | 9.9742 |

Wishing you All the Best
